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A tracer dispersion study of the drag-reduction effect in a turbulent pipe flow

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Remarkable differences in dispersion of a tracer material injected into turbulent pipe flows of water and water containing as little as 2·5 parts per million by weight of a soluble high-molecular-weight drag-reducing polyoxyethylene additive have been measured. Analysis of the tracer response curves in terms of a simple one-parameter model shows that the observed results are compatible with a drag-reduction mechanism based on thickening of the viscous sublayer adjoining the wall. Other experiments, reported briefly, suggest that polymer adsorption on to the wall is responsible for this thickening.

1. Introduction

More than twenty years ago, Toms (1949, pp. 135–141) published results showing that the frictional drag in turbulent tube flow could be significantly reduced by adding small amounts of certain soluble high-molecular-weight polymers to the flowing liquid. Recent discoveries of highly effective drag-reducing additives have led to numerous investigations of the drag-reduction effect in view of its possible practical applications in reducing the cost of pumping liquids, increasing the capacities of flow channels, reducing the frictional drag on naval craft, and the like. The research efforts and results available in this field have been reviewed and summarized from several points of view (Deavours 1966; Fabula, Lumley & Taylor 1966; Gadd 1965, 1966*a*; Hoyt 1968; Little 1969; Lumley 1967, 1969; McCullough 1969; Patterson, Zakin & Rodriguez 1969; Pruitt & Crawford 1965; Tulin 1966).

Up to now it has not been possible to make full practical use of the drag-reduction phenomenon since its mechanism remains unclear: its magnitude in a given flow situation cannot be predicted confidently without extensive preliminary experimentation. Many possible mechanisms of the phenomenon have been proposed, all of which are able to explain one or more but not all of its

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§ Present address: Technical Services Department, Alumina Partners of Jamaica, Spur Tree P.O., Jamaica manifestations; these have been discussed by Fabula *et al.* (1966) and by Lumley (1967, 1969). No definitive conclusion as to a single operative mechanism could be reached from the experimental data available. For convenience, the mechanisms which have been proposed can be summarized under three main headings:

(i) The viscoelastic hypotheses suggest that drag reduction is connected with viscoelastic behaviour of the polymer solutions. Many early drag-reduction studies were carried out with high polymer concentrations ($> 0{\cdot}5\,\%$ by weight), at which concentrations the solutions are indeed often measurably non-Newtonian or viscoelastic in their flow behaviour. However, with additives such as the high-molecular-weight polyoxyethylenes used in the present work, reductions in turbulent drag of up to 80 % may be obtained at concentrations below 50 parts per million by weight (ppm). At concentrations approaching 0·5 % by weight, our polyoxyethylene solutions had shear-rate-dependent viscosities, and appreciable normal stresses were measured on a Weissenberg Rheogoniometer; in addition, there were other manifestations of elastic behaviour, such as jet swelling and the ability of the solution to 'flow uphill', as described by James (1966). At the concentrations used in the drag-reduction tests described below (2·5–30 ppm), however, we could find no evidence of macroscopic normal stress effects or even of any departure from purely Newtonian viscous behaviour. Although Oliver & MacSporran (1968) report normal stress effects for solutions of a high-molecular-weight polyacrylamide as dilute as 10 ppm, these results are controversial in that normal stresses of the same order of magnitude were obtained for pure solvents (water, toluene, *n*-heptane). In a series of careful experiments, Gadd (1966*b*) and Brennen & Gadd (1967) showed that initially viscoelastic drag-reducing solutions could lose their elastic behaviour on aging without the drag-reducing properties being affected. As a result, hypotheses invoking macroscopic viscoelasticity of the solutions do not seem attractive. The solutions may, of course, still possess elastic properties in the sense that the individual macromolecules in solution may possess characteristic relaxation times, but hypotheses based on these properties are conveniently included under the second group below.

(ii) A second group of hypotheses is based on the assumption that the long-chain, randomly coiled polymer macromolecules in solution interact with the turbulent eddies either near the wall or in the core of the flow to modify the turbulence characteristics of the stream. For this type of mechanism to be operative, it seems that the suspended macromolecules need to be at least commensurate with the Kolmogorov microscale of the turbulent stream, i.e. with the eddies in the dissipative size range. Calculations by Virk *et al.* (1966, 1967) have shown that the coiled polymer molecules are usually at least an order of magnitude smaller than the eddy microscale, and it has therefore been necessary to postulate that there is shear-stretching of the macromolecular coils, tangling of many molecules to form larger units, or the like. Although these hypotheses can explain many aspects of the drag-reduction effect, recent work (Rubin 1970) has cast doubt on whether the structure changes sufficiently at high shear rates to explain the effect; so-called 'super-molecular aggregates' have been reported in aqueous carboxymethyl-cellulose solutions at very high concentrations (over 1 % by weight) by Barenblatt *et al.* (1966), but up to the present their presence in very dilute solutions has only been inferred.

(iii) The hypotheses in the third group are based on the assumption that the viscous sublayer in a pipe or channel is significantly altered (thickened) in the presence of the polymer additive, either by physical adsorption (El'perin, Smol'skiĭ & Levental 1965) to provide a resilient wall layer, or by the macromolecules interfering with the dual processes of eddy generation and ultimate eddy dissipation, both of which usually pass through maximum intensities near the edge of the viscous sublayer. The 'wall-slip' hypothesis used by Kozicki & Tiu (1968) to explain the drag-reduction phenomenon can be regarded as merely another way of expressing a sublayer thickening.

Results recently reported by Barenblatt & Kalashnikov (1968) suggest that in pipe flows two mechanisms may be present simultaneously, namely thickening of the viscous sublayer by the additive and also damping of eddies in the core of the flow by the elastic macromolecules, with the former playing the dominant part in dilute solutions.

At first sight, it seems that it should be a simple matter to measure turbulence intensities in the cores of the flows to determine if the group (ii) hypotheses are valid, and to measure distributions of the time-averaged local velocities to determine if the wall-neighbouring flow sublayer is thickened, as implied by the group (iii) hypotheses. Unfortunately, it has been found that all of the traditional measurement techniques are at least open to doubt or lead to difficulties of interpretation when applied to flows of polymer solutions. Heat-transfer characteristics are modified along with the flow behaviour, so that interpretation of hot-film anemometer measurements becomes difficult, as shown by Lindgren & Chao (1967), Smith *et al.* (1967) and Barenblatt, Kalashnikov & Kudin (1968), and in fact both increased and decreased turbulence intensities in polymer solutions (compared to the solvent only) have been reported by Spangler (1969), while the measurements of Seyer & Metzner (1969*a*) indicate essentially no change in turbulence intensity in the core of a pipe flow of the solution. Similarly, Astarita & Nicodemo (1969), Barenblatt & Kalashnikov (1968), Metzner & Astarita (1967), Savins (1965) and Smith *et al.* (1967), among others, have shown that Pitot (total-head) tubes may also give unreliable results in polymer solution flows, particularly when the probe diameter is small. While relatively large Pitot probes appear to give satisfactory results in very dilute solutions (Friehe & Schwarz 1969; Nicodemo, Acierno & Astarita 1969; Wetzel & Tsai 1968), it is impossible to explore the crucial region very close to the wall with them. Thus, while some velocity profiles have been obtained in drag-reducing polymer solutions, as will be discussed below, undisputable data on local mean velocities and turbulence characteristics in these solutions against which the various hypotheses can be tested are still very few as a result of these practical difficulties.

From the classical researches of Taylor (1953, 1954*a*, 1954*b*), Aris (1956, 1959) and others, and the more recent developments reviewed by Levenspiel & Bischoff (1963) and Taylor & Turner (1966), it is known that the degree of axial dispersion of an ideal pulse of tracer material introduced into a pipe flow depends on the shape of the velocity profile in the pipe (among other factors), and should be particularly sensitive to the presence of an appreciably thickened viscous sublayer or other layer adjacent to the wall if this existed in a turbulent flow. It therefore seemed that carefully performed *comparative* measurements of the dispersive axial spread of initially ideal tracer pulses in turbulent flows of dilute drag-reducing polymer solutions and of the pure solvent in the same pipe loop and at identical Reynolds numbers† would provide a powerful and rapid means for determining whether the stream structure near the wall is significantly altered in the presence of the polymer additive. The primary purpose of this paper is to report on the results of such tests, which have been made at several Reynolds numbers and concentrations of a selected polymer (polyoxyethylene) in water using the procedures and equipment described below, which we feel show clearly that under the conditions investigated there is a modification of the stream near the wall under drag-reducing conditions.

As is the case in most tracer studies of flow behaviour, the procedure used suffers from the drawback that the selection of a physical model of the actual processes occurring which will explain the overall observable result (the degree of dispersion of the tracer) is largely a matter of trial and error. However, we also present a preliminary and admittedly over-simplified model of the phenomenon, based partly on other experiments designed to determine the physical nature of the drag-reduction effect, which is capable of explaining at least qualitatively all of the experimental observations.

2. Experimental equipment and procedures

The apparatus used is shown schematically in figure 1. A detailed description is available elsewhere (Arunachalam 1969), so that only the features important in the present work need be discussed here. De-ionized water of very low electrical conductivity was pumped from a storage tank through a control valve (3), flow meter (5), and flexible hose (6) into the test section (14), which consisted of 20 ft of straight, hydraulically smooth 0·486 in. bore type 316 stainless-steel pipe fitted with a tracer injector (12), two conductivity cells (13, 19) for measuring the concentration of the sodium chloride used as the tracer material, and two sets of pressure tappings connected through piezometer rings at 15 and 16 to a sensitive differential pressure transducer (17), by means of which the pressure drop over the final section of the pipe could be measured and recorded continuously.

The tracer injector (12) was patterned after that described by Taylor (1954*b*) and is shown in figure 2. It consisted of a spring-loaded piston operating at right angles to the pipe axis. The piston contained two cavities of exactly the same

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† The density of the polyoxyethylene solutions used in this work (maximum concentration 30 ppm) was not measurably different from that of water. Viscosities of all the solutions (2·5–30 ppm) were measured on a Weissenberg Rheogoniometer over a very wide range of shear rates. No shear-rate dependence was detected at these concentrations, and the viscosities were very close to the viscosity of water. The Reynolds numbers reported in this work are therefore based on the properties of the solvent (water). The error involved is at most about 2 % (30 ppm solution), which is only slightly larger than the estimated inaccuracy in determining the total flow rates.

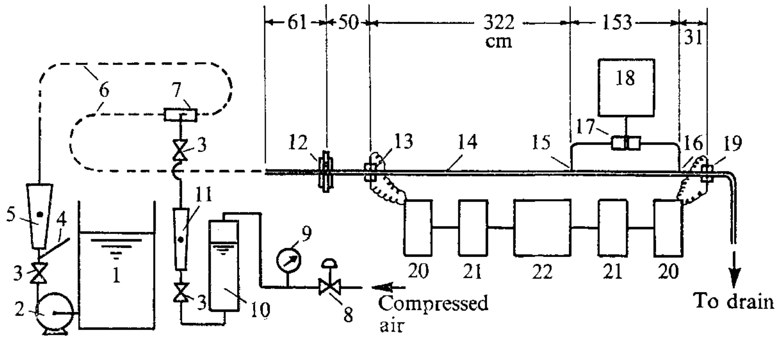

FIGURE 1. Flow diagram of equipment (dimensions in cm). 1, de-ionized water reservoir; 2, centrifugal pump; 3, valves; 4, mercury-in-glass thermometer; 5, water flow meter; 6, flexible hose; 7, polymer solution mixer; 8, pressure controller; 9, pressure gauge; 10, polymer solution reservoir; 11, polymer solution flow meter; 12, tracer injector; 13, upstream conductivity probe; 14, test section of pipe; 15, upstream static pressure tapping; 16, downstream static pressure tapping; 17, differential pressure transducer; 18, pressure drop recorder; 19, downstream conductivity probe; 20, conductivity bridges; 21, amplifiers; 22, high-speed two-channel oscillographic recorder.

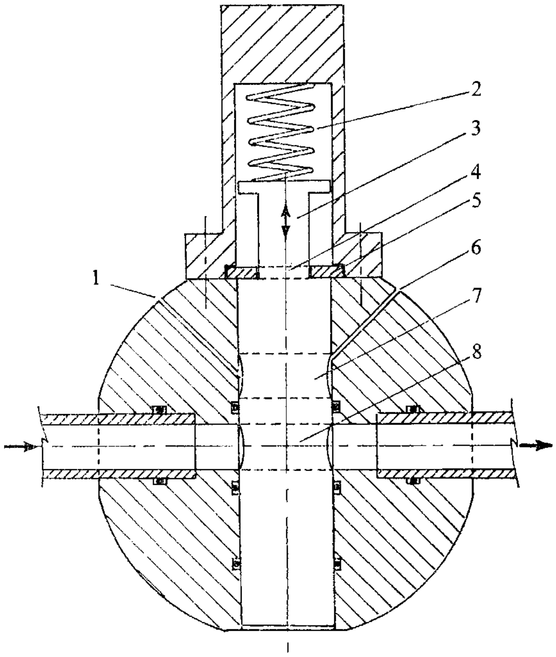

FIGURE 2. Tracer injector. All seals were effected by O-rings. 1, port for introducing tracer solution with hypodermic syringe; 2, spring; 3, piston travelling at right angles to pipe axis; 4, trigger slot; 5, trigger plate; 6, tracer solution overflow port; 7, chamber for tracer solution; 8, main flow chamber.

diameter as the inside diameter of the pipe. Leakage from one cavity to the other was prevented by a series of O-ring seals. In use, the lower cavity was aligned with the pipe axis and the flow was established through it. The upper cavity was filled with the tracer solution through the filling and overflow ports, and at the required instant was brought into line with the pipe axis by releasing a catch on the spring-loaded piston. In this way a uniform slug of tracer material covering the entire pipe cross-section could be introduced essentially instantaneously with a minimum of disturbance of the flow. Highly reproducible results were obtained in repeat runs. Any non-ideality in pulse shape was allowed for by measuring response curves at two positions along the pipe, as discussed later.

The conductivity cells consisted essentially of laterally insulated stainless-steel electrodes set diametrically opposite each other in a short section of polymethylmethacrylate tube machined to the same bore as the pipe and carefully aligned with the main axis of the working section. The exposed ends of the electrodes were practically flush with the pipe wall. It is obvious that only an average concentration across the cross-section can be measured in this way, but this procedure is merely an electrical analogue of the optical concentration measurement used successfully in such studies by Taylor (1953) and Elder (1959), and in any case we are mostly interested in purely comparative measurements with and without the polymer additive. This probe geometry offered the advantage of disturbing the flow as little as possible at the first measurement point.

The conductivity cells were connected to conductivity meters (20) (figure 1), amplifiers (21), and a high-speed light-writing two-channel recorder (Honeywell Visicorder) (22). Oscilloscope tests showed that the rate of response of the measuring and recording system was not a limiting factor in obtaining the tracer response curves. Preliminary experiments in which dilute sodium chloride solutions were circulated through the conductivity cells by pumping in a closed loop showed that the conductivity recorded was linearly related to the NaCl concentration up to tracer concentrations somewhat above the 0·5 % by weight (before dilution) which was used in the experiments, and that the background conductivity of the de-ionized water was negligibly small; both of these conditions must be satisfied if the final traces are to be interpreted directly.

The pressure tappings were carefully polished to remove all traces of burr, and checks were made with pure water flowing in the pipe that the pressure drops corresponded with the accepted values of the friction factor for smooth pipe flow. The differential pressure transducer and recorder were calibrated against standard U-tube manometers. All tests were carried out at temperatures between 19 and 23 °C, as measured at the time of the run on a thermometer (4, figure 1).

Polymer solution (0·5 % by weight in de-ionized water) was injected into the main flow of water from a pressurized reservoir (10) through a flow meter (11) and a mixing unit (7) well upstream of the working section. The dilute polymer solution was not recirculated. The concentration of the polymer in the dilute working solution after mixing was calculated from the settings on the flow meters (5) and (11), which were previously calibrated. For runs without polymer, the polymer solution flow was stopped and the system was flushed with de-ionized water until the pressure drop regained its accepted value for pure water for the given Reynolds number.

The polymer used in all the experiments reported here was 'Polyox WSR-301', a straight-chain polyoxyethylene manufactured and provided by the Union Carbide Corporation; the viscosity-average molecular weight was measured and found to be close to $4{\cdot}5 \times 10^6$. The 0·5 weight % master solution of the polymer was prepared in cold de-ionized water under stirring conditions selected and controlled to minimize shear degradation. Solution made from a single batch of polymer was used to eliminate possible effects of the known batch-to-batch variation of the polymer properties. The drag-reduction properties of Polyox WSR-301 solutions are known to be unaffected by additions of sodium chloride up to concentrations several thousand times the polymer concentration (White 1969), so that the dilute NaCl solution used as the tracer is most unlikely to have modified the behaviour of the system.

The salt tracer solution was prepared by dissolving analytical grade NaCl in de-ionized water and diluting to 0·5 % by weight.

In carrying out the tests the flow rates of the water and polymer solution were first adjusted to the required values. After the pressure-drop reading at 18 (figure 1) reached a steady value, the tracer solution ($\sim 1{\cdot}2\ \mathrm{cm}^3$) was injected into the flow essentially instantaneously by triggering the injector, and the conductivity response curves at the first and second conductivity cells were recorded on 22 (figure 1). The entire record (except when very severe tailing occurred at the downstream point) was complete in less than 10 sec from the time of injection; the chart speed at 22 was selected appropriately. The flow rates and temperature at the time of the test were recorded manually on the chart.

3. Evaluation of the experimental data

From the response curves recorded at the upstream and downstream conductivity cells, conductivity values (linearly proportional to the mean concentration) were read at time intervals of 0·01–0·04 sec along the time axis covering the entire response including the tail; this involved 100–400 values, depending on the degree of tailing. These data points were fed into a computer program which carried out the following steps. The curves were first normalized by dividing each 'concentration' value by the area under the response curve. The Fourier coefficients $(a_{1,n}, b_{1,n})$ and $(a_{2,n}, b_{2,n})$ to fit the normalized upstream and downstream responses were then computed, and from these the coefficients $(a_{0,n}, b_{0,n})$ of the true impulse response curve were calculated. Meaningful values of $a_{0,n}$, $b_{0,n}$ could be obtained for $n = 0$ to approximately 100. Noise effects began to appear only at values of n greater than about 85, corresponding to frequencies approaching the mains frequency used in the conductivity bridges.

In this way it was possible to generate the true impulse response curves even in the presence of possible tracer pulse input imperfections by using the measured responses at two positions along the pipe. From each impulse response curve the mean tracer residence time, T_m, and the spread of the residence times about the mean T_s^2, were calculated. The time base of each normalized impulse response curve was then rendered dimensionless by replotting the results in terms of the relative time $\theta = t/T_m$, where t is the actual time from injection of the pulse input. Following the nomenclature proposed by Danckwerts (1953), this exit age distribution function of the tracer will be termed the $E(\theta)$ curve. The $F(\theta)$ or integrated response curves are then easily obtained by numerically integrating the experimental $E(\theta)$ curves.

From the definition of the relative time, the value $\theta = 1$ characterizes the time taken for the fluid to flow from the point of injection of the pulse of tracer to the measurement point at the bulk mean velocity of the liquid in the pipe. On each of the $F(\theta)$ curves the value θ_i of the relative time θ at which $F(\theta)$ first became measurably greater than zero was obtained. Clearly, the value $\theta = \theta_i$ represents the relative time taken for the first detectable elements of the tracer to travel the same distance to the measurement point. If there were no lateral dispersion of the tracer material in the pipe, this would correspond to the time taken for fluid travelling at the maximum velocity in the cross-section to traverse the given length of pipe, and the value of θ_i would be numerically equal to the ratio $\epsilon' = \overline{U}/\hat{U}'$, where $\overline{U}$ is the bulk-average velocity and $\hat{U}'$ is the maximum velocity in the pipe cross-section, which is given by Rothfus *et al.* (1957) and Patel & Head (1969) as approximately 0·82 for fully developed turbulent pipe flows. Since some degree of lateral dispersion from the element of tracer travelling at the centre of the pipe will in fact occur, the concentration of this element may fall to so low a value before reaching the measurement point that its passage cannot be detected. Instead, the 'start' of the $E(\theta)$ or $F(\theta)$ curve along the θ-axis will correspond to the mean velocity $\hat{U}$ of a small but finite zone of fluid around the pipe axis, so that $\hat{U} < \hat{U}'$, and hence the measured value $\theta_i = \epsilon = \overline{U}/\hat{U}$ must be expected to be slightly larger than the true value ϵ'. Since the profile of the mean velocity in the centre of a fully turbulent pipe flow is essentially flat, however, no serious error will arise subsequently if $\hat{U}$ is interpreted as the 'maximum' velocity of the flow as long as the velocity profile in the central part of the flow in the polymer solution is similar in shape to that in the solvent at the same Reynolds number; this similarity will be discussed below.

In this way the value of the ratio ϵ was determined for each run, and the coefficient of variance $\gamma_1 = T_s/S_m$ and its square $\gamma = \gamma_1^2$ were calculated. The measured pressure drop was evaluated in the form of the friction factor f and the measured flow rate was expressed as the Reynolds number Re. (The Reynolds number was based on the physical properties of the solvent, for the reasons pointed out earlier.) Where necessary, the subscript w is used to denote values for pure solvent (water), and the subscript p for values in polymer solutions.

The values of f, T_m, T_s^2, γ, and ϵ obtained from the experimental results are summarized in table 1 for pure de-ionized water and Polyox WSR-301 solutions of several concentrations (2·5, 5, 10, 15, 20, and 30 ppm by weight) at several levels of the Reynolds number (Re approx. 22,000–62,000). The effects of changes of temperature from one run to the next ($< \pm 2$ °C) on viscosity were taken into

Reynolds no. (Re)	Polyox WSR-301 concentration (ppm)	Measured friction factor (f)	Mean tracer residence time (T_m [−])	Spread of residence times (T_s^2 [−])	Ratio of mean/'centre' velocities in pipe (ϵ)	Layer thickness, h (cm)	y at $y^+ = 10$ (cm × 10^2)	$\frac{h}{y\|_{y^+=10}}$
22,600	0	0·0056	2·91	0·030	0·88	—	1·03	—
22,700	2·5	0·0016	2·57	0·483	0·76	0·011	1·91	0·59
22,900	5	0·0015	2·52	1·670	0·73	0·018	1·96	0·89
22,200	10	0·0014	2·89	1·953	0·61	0·019	2·12	0·88
22,100	15	0·0014	3·39	3·080	0·52	0·021	2·19	1·00
22,900	20	0·0013	2·93	2·612	0·61	0·021	2·09	0·99
22,900	30	0·0014	3·37	3·902	0·50	0·024	2·05	1·17
27,500	0	0·0051	2·47	0·028	0·90	—	0·89	—
27,500	2·5	0·0016	2·43	0·262	0·76	0·009	1·59	0·58
26,300	5	0·0014	2·35	0·969	0·66	0·015	1·75	0·88
26,600	10	0·0013	2·57	1·479	0·61	0·017	1·81	0·95
26,700	15	0·0013	2·72	2·251	0·55	0·021	1·82	1·14
27,600	20	0·0012	2·61	1·462	0·60	0·018	1·79	0·98
27,800	30	0·0012	3·06	3·055	0·51	0·022	1·78	1·27
32,200	0	0·0052	2·04	0·009	0·89	—	0·75	—
32,100	2·5	0·0017	1·91	0·411	0·73	0·012	1·34	0·89
32,500	5	0·0013	1·86	0·527	0·73	0·013	1·47	0·89
31,200	10	0·0012	2·16	0·973	0·61	0·016	1·58	1·03
31,300	15	0·0013	2·21	1·362	0·60	0·018	1·57	1·17
32,200	20	0·0012	2·53	1·520	0·51	0·019	1·54	1·23
33,000	30	0·0012	2·09	1·398	0·63	0·019	1·51	1·25
37,200	0	0·0052	1·87	0·010	0·88	—	0·65	—
38,400	2·5	0·0019	1·57	0·136	0·78	0·009	1·05	0·81
38,900	5	0·0014	1·59	0·253	0·74	0·011	1·19	0·90
38,300	10	0·0013	1·74	0·448	0·65	0·013	1·25	1·06
38,500	15	0·0012	2·00	1·038	0·56	0·018	1·32	1·34
38,500	20	0·0012	1·98	1·095	0·59	0·018	1·30	1·36
39,000	30	0·0012	2·04	1·380	0·55	0·019	1·31	1·48
57,000	0	0·0048	1·21	0·0025	0·89	—	0·44	—
54,100	2·5	0·0022	1·20	0·025	0·88	0·005	0·69	0·74
54,400	5	0·0017	1·18	0·023	0·85	0·005	0·79	0·63
54,000	10	0·0013	1·13	0·036	0·78	0·006	0·89	0·69
54,400	15	0·0011	1·17	0·092	0·75	0·008	0·95	0·89
54,300	20	0·0012	1·33	0·171	0·68	0·010	0·94	1·11
54,800	30	0·0012	1·40	0·240	0·61	0·012	0·94	1·26
61,200	0	0·0047	1·13	0·0036	0·88	—	0·42	—
61,900	2·5	0·0023	1·04	0·0059	0·88	0·003	0·59	0·45
61,600	5	0·0017	1·05	0·059	0·81	0·007	0·68	1·08
62,300	5	0·0018	1·08	0·017	0·86	0·006	0·67	0·94
62,200	10	0·0013	1·16	0·109	0·74	0·009	0·78	1·16
62,500	15	0·0011	1·10	0·144	0·69	0·010	0·83	1·25
62,200	20	0·0011	1·17	0·164	0·65	0·011	0·84	1·28
61,000	20	0·0011	1·20	0·239	0·60	0·013	0·88	1·43

TABLE 1. Measured values and parameters calculated from tracer response curves

account in calculating the Reynolds number. Since a run was essentially complete in the time taken to record the two response curves ($<$ 10 sec) the change of temperature during a run was assumed to be negligible.

Examples of the $E(\theta)$ and $F(\theta)$ curves obtained by evaluating the experimental tracer response curves are shown in figure 3 (*a*) and (*b*) for a Reynolds number of 55,000. The curves at the other Reynolds numbers were similar in nature and are not illustrated.

4. Qualitative discussion of results

A qualitative picture of the results can be obtained from figure 3 (*a*) and (*b*). All the curves shown in this figure refer to the same Reynolds number ($Re =$ 55,000), the same flow geometry precisely, and the same tracer injection and measurement conditions, and differ only in the concentration of the Polyox WSR-301 additive. All the curves are normalized and plotted in terms of the dimensionless time θ, so that direct comparisons are valid.

The following two observations may be made directly from figure 3 (*a*) and (*b*); they are also valid for the results at other Reynolds numbers:

(i) In the absence of the polymer additive (curve 1) the $E(\theta)$ curve is bell-shaped and is approximately symmetrical about the mean residence time ($\theta = 1$) as predicted by the axial dispersion theory originally developed by Taylor (1954*a*) for turbulent flows in sufficiently long pipes. The corresponding $F(\theta)$ curve is of course S-shaped and is also approximately symmetrical about $\theta = 1$. However, as the polymer concentration increases from 2·5 to 30 ppm (curves 2–7) there is a dramatic increase in the degree of 'tailing' on the right of the curves, leading to highly unsymmetrical curves with lower peaks.

(ii) In the absence of the polymer additive the value of $\theta = \theta_i$ at which the response first deviates significantly from zero lies in the range 0·85–0·90. As pointed out earlier, the expected value of the ratio of the mean to the maximum velocity ($= \theta_i$) is about 0·82, but with the present technique the measured values should be slightly higher, as observed. However, as the polymer concentration increases it can be seen that θ_i moves steadily towards smaller values, reaching a value of 0·6 at 30 ppm, showing that there is a considerable change in the ratio of the mean to the maximum velocity in the cross-section in the presence of the additive.

The significant increase in tailing suggests immediately that there is an increase in the fraction of the flow near the wall which moves slowly: the part of the tracer material entering this zone on injection will appear at the exit cross-section only slowly, either by flowing all the way from the injection position to the measurement point in the zone of low velocity, or by first migrating from the thickened slowly moving region into the faster moving core.

The second observation shows that as the polymer concentration increases, $\overline{U}/\hat{U}$ decreases, or the 'maximum' velocity $\hat{U}$ increases for a given flow rate, as required by the continuity condition of the incompressible fluid if the first observation is correctly interpreted as indicating a thickening or reduction in the mean velocity of the flow zone near the pipe wall.

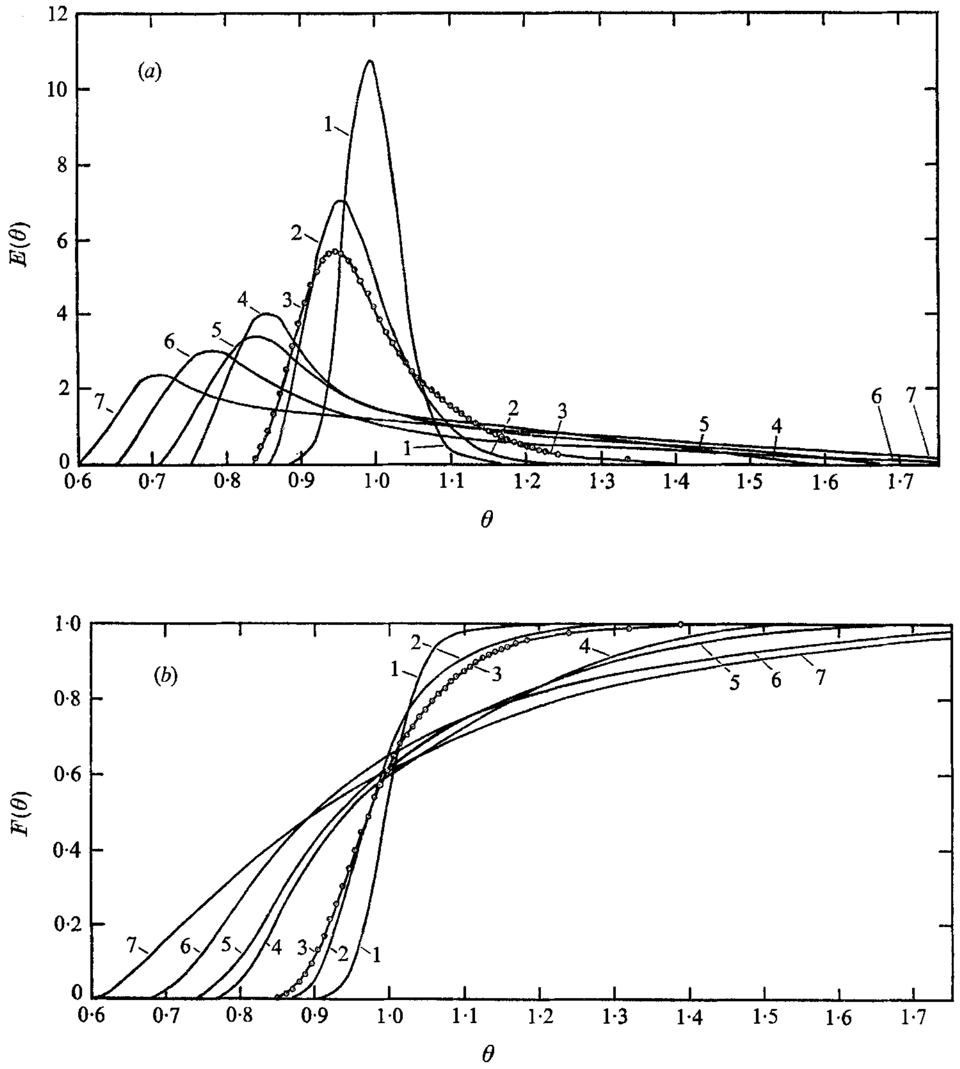

FIGURE 3. Normalized impulse response curves $E(\theta)$ and integrated impulse response curves $F(\theta)$ for water and aqueous Polyox WSR-301 solutions; Reynolds number = 55,000, θ = dimensionless time (defined in text). Data points are shown only for the 5 ppm solution for sake of clarity.

Curve	Solution concentration (ppm by weight)
1	0 (pure de-ionized water)
2	2·5
3	5
4	10
5	15
6	20
7	30

Both sets of observations are therefore consistent with a mechanism of the polymer action involving modification of the zone of flow near the pipe wall, at least for the polymer investigated. It is difficult to see how both observations can be interpreted without assuming such a mechanism, although of course the possibility of other mechanisms occurring in much more concentrated polymer solutions or solutions of other polymers cannot be ruled out. Qualitatively, therefore, the present interesting axial dispersion results support the modified wall-layer type of mechanism; an attempt at a more quantitative discussion is given below.

5. Discussion in terms of a quantitative model

Although the most immediately useful outcome of the present experiments is the fact that they support one of the three possible groups of mechanisms proposed for explaining the drag-reduction effect in the case of extremely dilute polymer solutions, as discussed above, the tracer response results appear sufficiently interesting to attempt fitting a model to the tracer results. The model presented below is a preliminary one, but in spite of the fact that it probably oversimplifies the physical phenomena occurring in some respects, it nevertheless yields results in reasonable agreement with those obtained by other methods.

(*a*) *Additional experimental observations*

In developing a model for the turbulent drag-reduction phenomenon in pipe flows of very dilute polymer solutions based on a wall-layer modification mechanism, the additional experimental observations outlined below have been used in order to justify some important physical simplifications made in the model. Complete details of these experiments are available elsewhere (Arunachalam 1969).

(i) Velocity profile measurements were made in the core regions of turbulent pipe flows of Polyox WSR-301 solutions (1–30 ppm) in a 1 in. diameter pipe using a relatively large impact probe; integration of the velocity profiles led to results in close agreement with the measured flow rates. When the velocities were plotted as the dimensionless velocity defect as a function of the dimensionless radial distance, the profiles in the central part of the pipe were very closely the same as those for the solvent only. The major difference was that the solution velocities in the central zone were slightly larger than in the case of the solvent flow: when the results were plotted in the form of the dimensionless velocity u^+ against the logarithm of the dimensionless distance from the wall y^+, the straight lines in the core region were parallel to the Prandtl line for water, but deviated upwards more and more as the polymer concentration increased. The so-called buffer zone also appeared to extend to greater values of y^+. This behaviour is in agreement with some of the rather scanty velocity profile data available in the literature (Elata 1966; Elata *et al.* 1966; Patterson & Florez 1969 (for polymer solutions); Rudd 1969; and Janberg 1970). Granville (1968) has provided a semi-theoretical explanation of this behaviour. On the other hand, Wells (1965), Patterson & Florez (1969) (for soap solutions), Nicodemo *et al.* (1969), Seyer & Metzner (1969*b*), and Squire, Castro & Costrell (1967) report that in drag-reducing fluids the turbulent velocity profiles are flatter than in the case of the solvent. In the first four cases, however, the polymer concentrations are three times to two orders of magnitude greater than the maximum concentration used in this work, which might explain some of the differences observed; the concentration at which the velocity profiles were measured is not reported in the fifth paper.

(ii) Adsorption measurements were made by passing Polyox WSR-301 solutions of known concentrations through beds of metallic powders or glass fibres and measuring the effluent concentration as a function of time using a turbidimetric technique (Arunachalam 1969).† Very clear break-through curves of the usual sort for batch-wise adsorption studies were obtained, indicating that this particular polymer does indeed adsorb on to high-energy solid surfaces such as metals and glass.

(iii) Samples of the liquid were removed during turbulent flow of the very dilute Polyox WSR-301 solutions in a 1 in. pipe from the centre of the pipe and through a wall-tapping. Concentration measurements by the technique in (ii) above showed that the concentration of additive in the wall layer was as much as twice as large as in the central part of the stream. Sampling lines were purged for a prolonged period to permit equilibrium adsorption to occur. Although strictly isokinetic sampling through a wall-tapping is impossible, the results indicated qualitatively that there appeared to be a wall-adsorption-type effect even under the dynamic conditions prevailing in a turbulent flow.

(iv) A few experiments were carried out in the tracer injection set-up used for the main experiments reported earlier in which pure water flowed in the pipe and a tracer slug of fairly concentrated Polyox WSR-301 solution in water was injected in place of the salt solution. The concentration was selected so that if the degree of axial dispersion of the tracer had been the same as in the case of the NaCl tracer in water the concentration at the peak of the response curve would have been 100 ppm or greater of polymer. Samples were taken at the downstream point as a function of time, and were subsequently analyzed for polyoxyethylene turbidimetrically. No peak whatever could be found. Instead, the effluent contained a long plateau of concentrations in the region of 5 ppm which continued to times of very many times the maximum of 10 sec or so required for the NaCl tracer to pass from the system even in the case of severe tailing. These results seemed to show clearly that the injected slug of polymer additive was

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† The concentrations were measured by precipitating the polyoxyethylene as a fine, stable suspension by adding under controlled conditions a solution of poly(acrylic acid) and a surface-active agent together with gum arabic as a protective colloid. The absorption of light of wavelength 425 nm by the unknown solution was compared with that by polyoxyethylene solutions of known concentrations prepared by diluting a 1 % master solution and treated identically to the unknown solution. Blanks of the water used were placed in the reference cell of the photometer, to which all the reagents were added in equal quantities. The calibration was repeated for each batch of reagents used and for each master solution of polyoxyethylene used in the experiments. Concentrations of less than 2 ppm could be detected.

rapidly adsorbed, and then desorbed from the wall only slowly. Since initially this behaviour was rather surprising, the tests were repeated carefully several times; subsequently it was found that this relatively slow desorption appears to be characteristic of polymers adsorbed on solid surfaces (see, for instance, the reviews by Ullman, Koral & Eirich (1957) and by LaMer & Healy (1963)).

(*b*) *Description of model*

Based on the experimental results described briefly above together with the findings from the main tracer dispersion tests, the following physical model has been developed. Since the profiles in the core of the pipe flows of very dilute polymer solutions appear to be similar to those in flows of the solvent alone at the same Reynolds number, apart from a shift to higher velocities, and since the various tests suggest that adsorption of the polyoxyethylene occurs on to the pipe wall, the simplest possible model appears to be one in which the adsorbed polymer is assumed to lead to the formation of an essentially stationary layer of some thickness h at the wall, outside of which the regular boundary layer develops. In other words, it is assumed that essentially all of the flow of the polymer solution occurs in a pipe whose radius is the true pipe radius reduced by the thickness of the 'stationary' layer h, leading to the observed higher centre-line velocities at a given flow rate. The increase in tailing of the response curves is explained by the increasing amount of tracer material in the essentially stationary layer as its thickness increases with increasing polymer concentrations. The model is admittedly over-simplified, since while the adsorbed polymer layer might be expected to considerably retard the flow, a discrete layer of zero velocity throughout its thickness is unlikely: physically, some sort of smooth transition at the edge of the zone affected by the adsorbed polymer seems more likely. However, it would be difficult to determine the parameters (at least two) of such a model from the experimental results (tracer response curves) with the necessary accuracy, while the single parameter of the present preliminary model (the layer thickness h) can be determined quite accurately, and it will be shown below that the values of h obtained are in quite reasonable agreement with the *increase* in sublayer thickness needed to explain the observed drag reductions. In other words, the model attempts to replace the thickened sublayer by an ordinary sublayer for the given flow rate plus the additional layer of thickness h, as indicated diagrammatically in figure 4(*a*) and (*b*).

There is also physical-chemical evidence in support of the postulated adsorbed layer of appreciable thickness. While the exact nature of polymer adsorption on to solid surfaces from solutions is still a matter of discussion, the various theories reviewed by Rowland *et al.* (1965) and Silberberg (1968) are in general agreement that only selected sites along the polymer chain become attached to sites on the solid surface during adsorption. The sections of the polymer chain between these sites hang loose from the wall in the form of loops, the length of which, it appears, may be at least as large as the radius of the randomly coiled chain away from the wall. A forest of such loops would presumably entrap the water near the wall, and lead to a relatively stationary zone. Such a mechanism has, in fact, been proposed already to explain very satisfactorily certain anomalous results observed in the capillary-tube viscosity measurements on certain polymer solutions (Öhrn 1955; Mineshita, Watanabe & Ono 1966; Tuijnman & Hermans 1957; van Oene & Cragg 1961), though it must be noted that the flow was not turbulent in these cases. The present model merely extends the idea to the more complex turbulent flow situation.

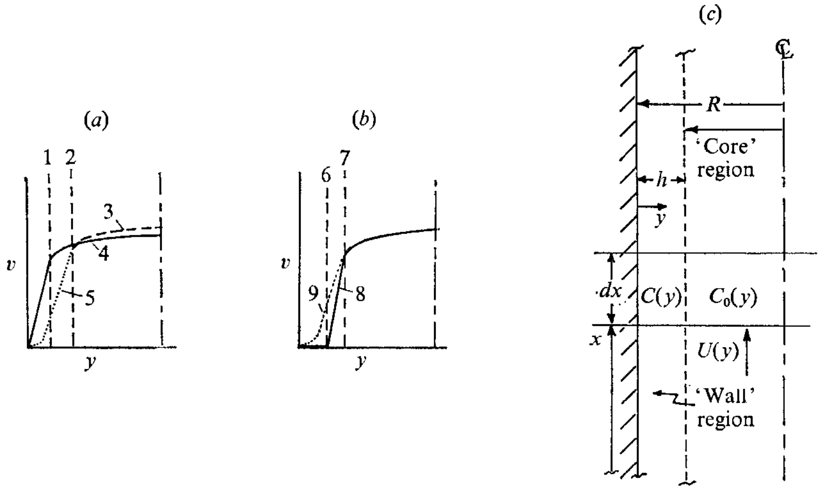

FIGURE 4. Sketch (not to scale) illustrating model and model parameters. (*a*) Turbulent velocity profiles in flow of water and polymer solution: 1, edge of viscous sublayer in water flow; 2, edge of viscous sublayer in solution flow; 3, measured part of velocity profile in core region of polymer solution flow (dashed line); 4, measured velocity profile in water flow at same Reynolds number as solution flow (solid line); 5, possible course of viscous sublayer profile in presence of adsorbed polymer at wall (dotted line). (*b*) Simplification introduced by model: 6, equivalent stationary layer of thickness h; 7, edge of viscous sublayer for polymer solution flow (corresponding to 2 in (*a*)); 8, viscous sublayer profile for water with same centre-line velocity as polymer solution flow, starting at distance h from wall (solid line); 9, possible viscous sublayer profile in polymer solution (as 5 in (*a*)) replaced by zero velocity in layer h and curve 8 in model (dotted line). (*c*) Sketch defining quantities used in setting up tracer mass balances.

(*c*) *Mathematical model*

To summarize, the following postulates are made: (i) the dispersion of tracer material in the entire pipe in the case of pure water as well as in the polymer solution, apart from a wall-layer of thickness h, can be described by the usual Taylor axial dispersion equation; (ii) transport of tracer material in the assumed wall-layer of thickness h in the polymer solution takes place solely by molecular diffusion in the direction normal to the tube wall; (iii) the depth of the 'adsorbed' wall-layer h is small compared with the tube radius.

In postulate (i) it should be noted that the Taylor axial dispersion analysis allows for the effects of the 'normal' laminar sublayer in a pipe flow. The additional effect of the 'adsorbed' part of the wall layer is allowed for in postulate (ii) in the case of the dilute solutions.

By mass balances over elements of length dx in the direction of flow, and on the basis of the three postulates above, we can write for pure water

$$D_w \frac{\partial^2 C_w}{\partial x^2} - \overline{U}_w \frac{\partial C_w}{\partial x} = \frac{\partial C_w}{\partial t}, \tag{1}$$

and for the polymer solution in the region outside the layer of thickness h

$$D_0 \frac{\partial^2 C_0}{\partial x^2} - \overline{U}_0 \frac{\partial C_0}{\partial x} - \frac{2D_m}{R}\left(\frac{\partial C}{\partial y}\right)_{y=h} = \frac{\partial C_0}{\partial t}, \tag{2}$$

where the third term allows for transfer of tracer material across the interface between the main flow and the adsorbed layer. In the adsorbed layer itself,

$$D_m \frac{\partial^2 C}{\partial y^2} = \frac{\partial C}{\partial t}. \tag{3}$$

Here C_w, C_0 represent the concentrations of the tracer material in water and in the polymer solution outside the layer of thickness h; D_w, D_0 are the Taylor axial dispersion coefficients of the tracer in water and in the polymer solution; D_m is the molecular diffusivity of the tracer material in the polymer solution close to the wall; R is the tube radius; t is the time; x, y are co-ordinates measured axially and radially inwards from the pipe wall; h is the thickness of the postulated adsorbed wall layer in the polymer solution ($h = 0$ for the solvent flow); C is the local instantaneous tracer concentration in the layer of thickness h.

In order to develop a theoretical expression for the model parameter (h) in terms of measurable quantities, the above equations may be solved with appropriate boundary conditions to yield the following moments of the external tracer age distribution functions:

$$T_{mw} = \frac{L}{\overline{U}_w}, \quad T_{mp} = \frac{L}{\overline{U}_p} = \frac{L}{\overline{U}_0}\left(1 + \frac{2h}{R}\right), \tag{4}$$

$$T^2_{sw} = \frac{2LD_w}{(\overline{U}_w)^3}, \quad T^2_{sp} = \frac{2LD_0}{(\overline{U}_0)^3}\left(1 + \frac{2h}{R}\right)^2 + \frac{4}{3}\frac{Lh^3}{\overline{U}_0 D_m R}, \tag{5}$$

where L is the known length of the test section.

We now define

$$\epsilon_w = \overline{U}_w/\hat{U}_w, \quad \epsilon_p = \overline{U}_p/\hat{U}_p, \quad \epsilon_0 = \overline{U}_0/\hat{U}_0, \tag{6}$$

$$\gamma_w = T^2_{sw}/T^2_{mw}, \quad \gamma_p = T^2_{sp}/T^2_{mp}, \quad \gamma_0 = 2D_0/\overline{U}_0 L, \tag{7}$$

where the subscripts w, p and 0 refer respectively to quantities for the flow of the pure solvent (water), to the flow of the polymer solution, including the layer at the wall of thickness h, and the flow of the polymer solution, excluding the part in the layer of thickness h.

By combining equations (4), (5) and (7),

$$T^2_{sp} = \gamma_0 T^2_{mp} + \frac{4}{3}\frac{Lh^3}{\overline{U}_0 D_m R} \tag{8}$$

and then from (7),

$$h^3 = \frac{3}{4}\frac{\overline{U}_0 D_m R T^2_{mp}}{L}(\gamma_p - \gamma_0). \tag{9}$$

Based on the core velocity profile measurements discussed earlier, we assume that the mean profile in the main part of the polymer solution flow, apart from the layer of thickness h, is similar in shape to that for pure water flowing with the same maximum (centre-line) velocity. We can therefore introduce two important simplifications:

$$\epsilon_0 = \epsilon_w \quad \text{and} \quad \gamma_0 = \gamma_w \tag{10}$$

(ϵ_w, γ_w being evaluated when $\hat{U}_w = \hat{U}_p$). Hence, $\overline{U}_0 = \epsilon_0 \hat{U}_0 = \epsilon_w \hat{U}_p$ (since $\hat{U}_0 = \hat{U}_p$), so that

$$\overline{U}_0 = \epsilon_w \overline{U}_p / \epsilon_p. \tag{11}$$

Finally, from (9) and (11),

$$h^3 = \tfrac{3}{4}(\gamma_p - \gamma_w)\,(\epsilon_w/\epsilon_p) T_{mp} D_m R. \tag{12}$$

Here γ_p, γ_w, ϵ_p, ϵ_w and T_{mp} are readily obtained from the tracer tests, D_m is a physical property of the system, and R is fixed by the tube geometry. Values of h calculated according to equation (12) for the various tracer tests are shown in table 1.

In deriving equation (12) it is assumed that $\epsilon_0 = \epsilon_w$ when ϵ_w is evaluated at $\hat{U}_w = \hat{U}_p = \hat{U}_0$. In fact, the tracer tests showed that for all practical purposes ϵ_w was constant over the range of Reynolds numbers investigated, so that it was possible to evaluate ϵ_0 at the same Reynolds number for the water as for the polymer run, thereby avoiding interpolation between measured values of ϵ_w. Values of ϵ_w at various Reynolds numbers obtained from a separate series of tracer experiments are shown in table 2, from which it is seen that $\epsilon_w = 0{\cdot}89 \pm 0{\cdot}03$ for the range $Re = 22{,}000$–$65{,}000$.

Reynolds no. (Re)	Mean tracer residence time (T_m [−])	Spread of residence times (T_s^2 [−])	Ratio mean/'centre' velocities in pipe (ϵ_w)	Measured friction factor (f)
21,800	2·95	0·021	0·90	0·0056
22,610	2·91	0·030	0·88	0·0056
26,200	2·65	0·022	0·89	0·0055
27,500	2·47	0·028	0·90	0·0051
28,600	2·41	0·015	0·91	0·0053
31,200	2·22	0·012	0·90	0·0053
32,200	2·04	0·009	0·89	0·0052
33,700	2·06	0·013	0·90	0·0052
36,900	1·74	0·011	0·89	0·0051
37,150	1·87	0·010	0·88	0·0052
40,450	1·72	0·011	0·89	0·0051
46,600	1·41	0·0032	0·91	0·0050
48,800	1·42	0·0088	0·86	0·0049
57,000	1·21	0·0025	0·89	0·0048
58,200	1·12	0·0020	0·92	0·0048
59,400	1·08	0·0035	0·90	0·0047
61,200	1·13	0·0036	0·88	0·0047
65,350	1·07	0·0048	0·89	0·0047

Table 2. Tracer results for pipe flows of pure water

(*d*) *Discussion of model and results*

The two important assumptions made in the model used for analyzing the results were that (i) the additional thickness h of the 'adsorbed' wall-neighbouring layer arising in the polymer solutions is small compared with the tube radius, and (ii) the tracer material (NaCl) is transported in this layer by molecular diffusion with a constant and known diffusivity D_m.

Geometrically, the first assumption appears justified in view of the small values of h which are calculated from the model (table 1). If this layer of thickness h were part of the normal boundary layer, it is realized that a considerable part of the total change of velocity between the wall and the centre of the pipe would occur in this layer, destroying the basis for the model. Since an adsorption effect at the wall in the polymer solutions and an increase in concentration near the wall are indicated by the experiments reported earlier, however, it is felt that the model, while involving obvious over-simplification of the true behaviour near the wall, is not unreasonable.

A similar argument may be used in support of the second assumption: the free-hanging loops of the adsorbed polymer molecules might be expected to lead to a considerable damping of the turbulent eddies which would normally be expected to penetrate within the distance h of the wall. At the same time, since the experimental data point to an increase in polymer concentration near the wall, it might be argued that the molecular diffusivity of the tracer is unlikely to remain constant as the wall is approached due to the increased concentration of the substrate. However, experimental work by Astarita (1965), Clough *et al.* (1962), Hirose (1969), and Hoshino & Sato (1967) on the diffusivities of various materials (including NaCl) in aqueous high-polymer solutions makes it clear that the diffusivities of small molecules in such solutions differ little if at all from their diffusivities in pure water at the same temperature even at polymer concentrations several hundred times larger than the bulk concentrations used in this work. It therefore seems reasonable to assume that $D_{m(\mathrm{NaCl-H_2O})}$ is constant in the dilute polymer layer of thickness h and has the same value as in water at the temperature of the experiment, which was obtained from the tabulated data in Washburn (1929).

We might also mention in this connexion some additional work which was done on a more complex model in which two independent parameters were determined from the tracer response results, the thickness of the 'adsorbed' layer h, and the mean effective diffusivity of the tracer in the layer of thickness h. The latter quantity unfortunately proved to depend rather critically on a higher power of the *difference* between ϵ_w and ϵ_p, which could not be determined with high accuracy even though the values of ϵ_w and ϵ_p could be determined individually with reasonable accuracy. As a result, the values for the mean effective diffusivity scattered rather badly. However, they were of the same order of magnitude as the molecular diffusivity, and showed no trend with increasing polymer concentration or with increasing Reynolds number, which was taken as an additional bit of evidence that it is suitable to use the molecular diffusivity in the simple one-parameter model.

Turning to the values of h in table 1 which were calculated using equation (12) on the basis of the simple model, it can be seen that (i) the value of h is small in all cases compared with the pipe radius ($\sim 0{\cdot}62$ cm), as required by the model assumptions; (ii) at a given polymer concentration, h decreases as the Reynolds number increases, and hence as the wall shear stress increases; (iii) at a given Reynolds number, h increases very rapidly initially as the polymer concentration increases from zero; further increase in h occurs only slowly above the very low concentration of 10 ppm.

These observations appear to be qualitatively in agreement with the adsorbed wall-layer thickening mechanism of the drag-reduction effect. The fact that thickened or adsorbed wall layers in the presence of drag-reducing polymer additives have not been detected experimentally in the past may be explained by the smallness of h, which is seen to be of the order of 0·1 mm. For instance, Goren (1966) and Goren & Norbury (1967) withdrew and analyzed samples of polymer solution from various radial positions in a turbulent pipe flow and reported a constant radial polymer concentration distribution under drag-reducing conditions, in contrast to our results mentioned earlier. However, it should be noted that in their experiments no samples appear to have been taken closer than about 1 mm from the wall. In fact, none of the usual quantitative measurement methods used in turbulent flows will readily give reliable results as close as 0·1 mm to a pipe wall even in the absence of complications due to the presence of polymer molecules. For this reason, the tracer technique used in the present work appears to be a powerful one of considerable value in studying complicated types of flow behaviour.

In the model, the entire 'adsorbed' layer at the wall is assumed stationary. Even though the free-hanging loops of the polymer molecules might be expected to greatly reduce the flow in such a layer, it is physically unlikely that the layer will have a sharply defined outer boundary at a distance h from the wall. Nevertheless, it seems justified to characterize the unknown and somewhat indefinite thickness of the true layer by means of the equivalent stationary thickness of the layer, h, as done in the model,† rather as the thickness of a boundary layer can be characterized in terms of a momentum thickness (Schlichting 1960, p. 123). If the model is to be considered valid, however, the values of h which it yields should clearly be of the same order of magnitude as the *increase* in thickness of the boundary layer which would be required to explain the drag-reduction effect quantitatively, but should be smaller than the true increase, since h represents the 'effective' thickness only.

To characterize the 'boundary-layer thickness' in a pipe, let us use a value of the dimensionless distance from the wall of $y^+ = 10$, which is close to the intersection ($y^+ \cong 10{\cdot}6$) of the extrapolated curve $u^+ = y^+$ (viscous sublayer

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† It is hoped to use a more elaborate model in the future to reinterpret the present tracer data. Aris (1959), for instance, has already developed the mathematics of a very elaborate model involving separate position-dependent velocities and effective molecular diffusivities in the core and wall zones of a flow, allowing for interchange of tracer between the zones. The difficulty in applying so complex a model lies in evaluating all the parameters involved with the necessary accuracy from the measured tracer response curves alone.

profile) and the Prandtl logarithmic law curve $u^+ = 5{\cdot}5 + 5{\cdot}75 \log y^+$ as given by Kay (1957), ignoring the usual 'buffer zone'. With the usual definitions of y^+, the friction factor f, and the Reynolds number, it is easily shown that

$$y^+ = (y/d)(\tfrac{1}{2}f)^{\frac{1}{2}}\, Re,$$

where y is the actual distance from the wall and d is the tube diameter. By inserting the numerical values $y^+ = 10$, $d = 1{\cdot}23$ cm, and the measured values of f and Re for each polymer solution run, the actual distance y from the wall corresponding to $y^+ = 10$ (the assumed 'boundary-layer thickness') has been calculated for each run. The values are given in the penultimate column of table 1. The last column of table 1 gives the ratio of h, obtained from equation (12), to this quantity. It can be seen that the ratio $h/y|_{y^+=10}$ is always of the order of unity, indicating that the total wall layer (adsorbed layer plus viscous sublayer) of the model is approximately twice as thick as the expected 'boundary layer' would normally be. This is in reasonable agreement with the prediction by Elata, Lehrer & Kahanovitz (1966) from extrapolations of Pitot-tube core velocity profiles that the 'edge' of the boundary layer moves from $y^+ \simeq 11$ to $y^+ \simeq 35$ (i.e. the 'boundary-layer' thickness increases by a factor of three, instead of a factor of two) under drag-reducing conditions. The thickening h obtained here is also of the same order as that shown by Kozicki & Tiu (1968) to be necessary if the drag-reduction data reported in the literature were to be formally explained by a 'wall-slip' type of mechanism.

Even the simple model used to analyze the present experimental tracer response results is therefore seen to yield values of the wall-layer thickness which are in general agreement with those obtained from quite different approaches, indicating the usefulness of the tracer technique in such studies.

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